Cork Institute of Technology

Bachelor of Engineering (Honours) in Mechanical Engineering- Stage 2

(EMECH_8_Y2)

Summer 2008

Mathematics

(Time: 3 Hours)

Answer **FIVE** questions.
All questions carry equal marks.

Examiners: Mr. P. Clarke
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Mr. T O Leary

1. (a) Solve the differential equation

$$\frac{dy}{dx}+3y=12 \qquad y(0)=1.$$

By using the Three Term Taylor Method with a step of 0.1 estimate the value of y at x=0.1. Calculate the error in this approximation. (8 marks)

(b) Find the general solution of the differential equation

$$\frac{d^2i}{dt^2}+4\frac{di}{dt}+4i=50\cos 3t$$

Express the steady state part of the solution as a single periodic function of the form $R\sin(3t-\alpha)$. Write down the maximum and minimum values of this function and find the smallest positive values of t for which these extreme values hold. (8 marks)

(c) Show that the expression

$$(6x^2-6y)dx-(6x-8y)dy$$

is the total derivative of a function. Find this function. Hence solve the exact differential equation

$$(6x^2-6y)dx-(6x-8y)dy=0 \qquad y(1)=2.$$

(4 marks)

2. (a) Find a Taylor Series expansion of the function $f(x,y)=x\ln(x-3y)$ about the values $x=4, y=1$. The series is to contain terms deduced from second order partial derivatives.

(7 marks)

(b) Find the partial derivatives of u and v with respect to x and y where

$$u=\sin^{-1}\left(\frac{2y}{x}\right) \qquad v=\sqrt{x^2-4y^2}\ .$$

(i) If $T=f(u)$ is an arbitrary function in u show that

$$x\frac{\partial T}{\partial x}+y\frac{\partial T}{\partial y}=0\,.$$

(ii) Estimate the value of v if the values of x and y were estimated to be 5 and 2 with maximum errors of 0.06 and 0.03, respectively. (8 marks)

(c) Find the minimum value of $V=x^2+2xy$ where $y=6+x$. Eliminate one of the variables and use a Lagrangian Multiplier. (5 marks)

3. (a) Find the Inverse Laplace transform of the expressions

(i) $\dfrac{20}{(s+1)(s^2+4)}$ (ii) $\dfrac{4s+8}{(s-1)(s-3)^2}$ (10 marks)

(b) By using Laplace Transforms solve the differential equation

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=16e^t \qquad y(0)=y'(0)=0$$ (6 marks)

(c) Find the zero and the poles of the transfer function $\dfrac{L[y]}{L[f(t)]}$ where

$$\frac{d^2y}{dt^2}+7\frac{dy}{dt}+40y+34\int_0^t y\,dt = f(t) \qquad y(0) = y'(0) = 0$$ (4 marks)

4. In answering the following question you are required to use the Method of Undetermined Coefficients. No marks will be awarded if any other method is used.

A mass is attached to a spring and a dashpot. The displacement x of the mass at any instant t is found by solving the differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t) \qquad x(0) = x'(0) = 0 .$$

(a) Solve this differential equation where m=1, c=4, k=20 and f(t)=40. (7 marks)

(b) Solve this differential equation where m=2, c=12, k=16, f(t)=64t (6 marks)

(c) Find the general solution of this differential equation where

m=1, c=0, k=-4, f(t)=20e^{2t} (7 marks)

5. (a) By using a method of your own choice solve for x where

$$\frac{dx}{dt} = -2x + y \qquad x(0) = 6$$

$$\frac{dy}{dt} = 5x + 2y \qquad y(0) = 0$$

By using a second method find the general solution for x and for y. (9 marks)

(b) Green's Theorem states: If C is a piecewise smooth closed curve that encloses a region R and if f(x,y) and g(x,y) have continuous partial derivatives throughout R then

$$\oint_C f(x,y)dx + g(x,y)dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)dA$$

where the direction of C is anticlockwise.

(i) Verify Green's Theorem where f(x,y)=4y^2, g(x,y)=24xy and R is the triangular region with vertices (-1,0), (1,0) and (1,4)

(ii) By summing horizontally find the second moment of area of this region about the x-axis. (11 marks)

6. (a) A force $\mathbf{F}=6x^2\mathbf{i}+24xy\mathbf{j}$ moves a mass the perimeter of the semi-elliptical region

$$\frac{x^2}{4}+\frac{y^2}{9}=1 \qquad x\geq 0$$

Find the work done by the force.

By evaluating a double integral locate the centroid of this region. (9 marks)

(b) The region R is the sector of the circle $x^2+y^2=5$ lying in the first and fourth quadrants with vertices (0,0), (1,2) and (1,-2).

(i) Evaluate the line integral

$$\oint_C 2y\,dx + 4x\,dy$$

where C is the perimeter of this region. The direction of C is anticlockwise.

(ii) If V is the volume with a constant cross section that is described by the region R above and if $0\leq z\leq 2$ then evaluate the triple integral

$$\iiint_V 6xz\,dV$$ (11 marks)

7. (a) A variate can only assume values between x=0 and x=2. The probability density function is given by

$$p(x)=A(3x^2+1).$$

Find the value of A, the mean value of the distribution and the median value of the distribution correct to two places of decimal. This value is close to x=1.5. (8 marks)

(b) A particular chemical is packed into bags with a mean content of 5kg and a standard deviation of 0.015kg. What percentage of bags contain (i) more than 5.02kg and (ii)between 4.98 and 4.99kg of chemical? EU regulations demand that a minimum content be printed on each bag and not more than 0.2% of bags are to weigh less than this minimum content. Find this minimum content. (6 marks)

(c) In a manufacturing process items are packed into batches of 50 and for a number of these batches the number of defective items were counted.

Number of defectives	0	1	2	3	≥ 4
Number of batches	70	21	8	1	0

Calculate the average number of defective items per batch. Calculate the probability that a batch of 200 of these items contains at most two defective items. Use both the Binomial and the Poisson distributions. (6 marks)

f(x)	**f′(x)** a=constant
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2-x^2}}$
uv	$u\frac{dv}{dx}+v\frac{du}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

f(x)	$\int$ **f(x)dx** a=constant
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
e^{ax}	$\frac{1}{a}e^{ax}$

LAPLACE TRANSFORMS

For a function f(t) the Laplace Transform of f(t) is a function in s defined by

$$F(s) = \int_0^\infty e^{-st} f(t)dt \quad \text{where } s>0.$$

f(t)	F(s)
A=constant	$\frac{A}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
sinhkt	$\frac{k}{s^2-k^2}$
coshkt	$\frac{s}{s^2-k^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at}f(t)$	F(s-a)
$f'(t)$	sF(s)-f(0)
$f''(t)$	$s^2F(s)-sf(0)-f'(o)$
$\int_0^t f(u)du$	$\frac{F(s)}{s}$
$\int_0^t f(u)g(t-u)du$	F(s)G(s)
U(t-a)	$\frac{e^{-as}}{s}$
f(t-a)U(t-a)	$e^{-as}F(s)$
$\delta(t-a)$	e^{-as}

Note: $\cosh A = \frac{e^A + e^{-A}}{2}$ $\qquad$ $\sinh A = \frac{e^A - e^{-A}}{2}$